

Theorem 1.1. Let G be a graph with n vertices and m edges. Then the chromatic number of G is at most $n/m + 1$.

Proof. Let G be a graph with n vertices and m edges. Then the average degree of G is m/n . Let v be a vertex of G . Then the degree of v is at most $m/n + 1$. Let G_v be the graph obtained by removing v and its incident edges from G . Then G_v has $n-1$ vertices and $m - \deg(v)$ edges. By induction, the chromatic number of G_v is at most $(n-1)/(m - \deg(v)) + 1$. Then the chromatic number of G is at most $(n-1)/(m - \deg(v)) + 1 + 1 = n/m + 1$.